AN ACCURATE FET MODELLING FROM MEASURED S-PARAMETERS

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Abstract

A new modelling algorithm has been developed which extracts a small-signal microwave equivalent circuit of an FET from measured S-parameters. The combinations of properly selected circuit elements with an S-parameter to be least-squares fitted greatly improve the accuracy and consistency of modelling. The absolute accuracy of determining element values was estimated to be better than 3 %.

I. Introduction

There exists a number of ways to extract a small-signal microwave equivalent circuit of an FET from measured data [1] - [2], among which most commonly used is a least-squares fitting of measured S-parameters to those of an FET equivalent circuit. A conventional modelling routine, which tries to fit all the S-parameters simultaneously in a least-squares sense using a single error function, usually neglects the difference in standard deviations of errors in individual measured S-parameters. It also tends to suffer from a so-called local minimum problem due to poor sensitivity of the error function to some circuit elements.

The new modelling algorithm, proposed in this paper, eliminates both problems by separate use of four S-parameters to each of which only a selected group of circuit elements is optimized to make a least-squares fitting of a model.

II. Description of the New Modelling Algorithm

An FET model is extracted by a least-squares fitting of measured S-parameters to those calculated from an FET equivalent circuit such as shown in Fig. 1. An iterative method is employed to solve a set of nonlinear equations which describes the calculated S-parameters. In each iteration cycle the equations are linearlized around estimated values of the circuit elements. Each iteration cycle of the new modelling routine is divided into eight consecutive optimization steps in each of which only a group of selected elements in the FET equivalent circuit is optimized to fit a specific S-parameter over a specific frequency range. This routine, thus, involves eight error functions to be minimized, as opposed to a single error function in the conventional routine. This partitioning improves the accuracy of modelling significantly as will be discussed in the next section. Table 1 summarizes the combinations of selected elements

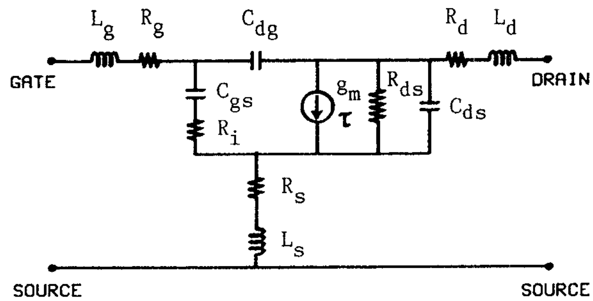

Fig. 1: An FET equivalent circuit

optimized with an S-parameter and a frequency range fitted in each step. A frequency range of measurement is typically from 1.5 GHz to 26.5 GHz. In the first step of an iteration cycle only R_{ds} and C_{ds} are optimized to fit S_{22} of the FET model to the measured S_{22} over the entire frequency range of measurement. The other elements are fixed in value to those obtained in their last optimization cycle. In the second step only C_{gs} is optimized to fit the calculated S_{11} to the measured S_{11} in the entire frequency range and the subsequent optimization steps follow as shown in the table. The order of the eight steps is important to enhance the numerical stability and the rate of convergence of the modelling algorithm [3][4]. An entire modelling routine requires iteration of this eight-step optimization cycle.

Table 1: Eight steps in an iteration cycle of the new modelling routine

Step	Elements optimized	S-parameter fitted	Freq. Range
1	R_{ds}, C_{ds}	S_{22}	all
2	C_{gs}	S_{11}	all
3	C_{dg}, R_s	S_{12}	all
4	g_m	S_{21}	all
5	R_d, L_d	S_{22}	upper half band
6	R_g, R_i, L_g	S_{11}	"
7	L_s	S_{12}	"
8	τ	S_{21}	"

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III. Theoretical Background

When the S-parameters of an FET are measured at N discrete frequency points, we have n (= 4N) measured data to be fitted to the S-parameters calculated from an FET equivalent circuit which contains q circuit elements.

A least-squares fitting procedure associated with an error function can be then generally described by the following normal equation which is linearlized around estimated values of the circuit elements in each iteration cycle of an optimization routine [3].

$$\mathbf{A}^{*T}(\mathbf{m} - \mathbf{A}x) = 0 \tag{1}$$

$$\text{where } \mathbf{m} = [\, m_i \,] = [\, \frac{S^i_M - S^i_X \big|_{\mathbf{x}=0}}{\delta_i} \,] \quad \text{for} \begin{cases} i=1,\ldots,n' \leq n \\ j=1,\ldots,q' \leq q \end{cases}$$

$$\mathbf{A} = [\, a_{ij} \,] = [\, \frac{\partial s_i}{\partial x_j}\Big|_{\mathbf{x}=0} \,]$$

$$\mathbf{x} = [\, x_j \,] = [\Delta C_{gs}, \Delta C_{ds}, \ldots, \Delta g_m, \ldots, \Delta\tau]^T$$

$$\text{and} \quad s_i = S^i_X / \delta_i$$

$\mathbf{m}$ is a column vector, each element of which represents a normalized difference of an S-parameter measured at a frequency, S^i_M, from the one calculated, S^i_X, for the estimated values of the circuit elements. δ_i, being usually unknown or difficult to estimate, is a standard deviation of measurement error in S^i_M. $\mathbf{x}$ designates a column vector of element-value correction required from the estimated values. An element, a_{ij}, of an n' x q' matrix, $\mathbf{A}$, represents the sensitivity of a normalized S-parameter, s_i, to an element correction, x_j. The error function is thus described as the 2-norm of a vector, ($\mathbf{m} - \mathbf{A}x$). An asterisk * denotes a complex conjugate, a superscript T indicating a transposed matrix. The notation $n' < n$ is used to emphasize the fact that all the measured data is not necessarily used in constructing a normal equation. The $q' < q$ means that in general not all the elements are optimized simultaneously using a single normal equation.

The variance, $\delta_{x_j}^2$, of the optimized value of a circuit element, caused by the variance, δ_i^2, of measured data, can be estimated by expressing x_j as a linear combination of m_i's in Eq. (1) [5].

$$\boldsymbol{\delta}_x^2 = [\, \delta_{x_1}^2, \ldots, \delta_{x_j}^2, \ldots, \delta_{x_{q'}}^2 \,]^T$$

$$= \frac{1}{|\mathbf{B}|} [\, |\mathbf{B}_{11}|, \ldots, |\mathbf{B}_{jj}|, \ldots, |\mathbf{B}_{q'q'}| \,]^T \tag{2}$$

$$\text{where } \mathbf{B} = \mathbf{A}^{*T}\mathbf{A} = [b_{ij}] = [\sum_p \frac{\partial s_p^*}{\partial x_i} \cdot \frac{\partial s_p}{\partial x_j}\,]$$

$|\mathbf{B}|$ is a determinant of $\mathbf{B}$ and $\mathbf{B}_{jj}$ represents a (q'-1) x (q'-1) matrix obtained by deleting the j-th row and the j-th column of $\mathbf{B}$. The highest accuracy of determining x_j is achieved when $\delta_{x_j}^2$ is minimized by reducing the degree of the singularity of matrix $\mathbf{B}$ and by making b_{jj} dominant among all the b_{ij}'s. This implies that if more than one element are simultaneously optimized using a single normal equation, a matrix $\mathbf{B}$ should be as close as possible to a diagonal matrix with all the diagonal elements of comparable magnitude. This scheme also helps reduce errors generated in numerical calculations due to rounding-off or truncation [3]. The simultaneous optimization of minor circuit elements with major elements, as is done in the conventional optimization routine, significantly deteriorates the modelling accuracy by increasing the relative magnitude of $|\mathbf{B}_{jj}|$ to $|\mathbf{B}|$.

The new optimization algorithm, proposed in the previous section, minimizes the variance, $\delta_{x_j}^2$, by optimizing a group of properly selected circuit elements to fit an S-parameter most sensitive to them. The separate use of four S-parameters also avoids ambiguity in estimating the variances of measured data, which otherwise should be taken into consideration in Eq. (1). The combinations listed in Table 1 were chosen as a result of intensive experimental investigation of S-parameter sensitivity performed on practical FETs, since an analytical approach seemed impractical.

IV. Experiments

The new modelling algorithm has been implemented in a software using the Super-Compact gradient optimizer.

1) Modelling Accuracy

The absolute accuracy of the new algorithm was estimated by performing an optimization on fictitious measured data. The data was generated for a frequency range of 1.5 GHz to 25 GHz by calculating S-parameters from an FET equivalent circuit which simulates a 0.5 x 300 um MMIC FET. Optimization using the new modelling routine was performed on this fictitious measured data, the results of which should converge to the original FET equivalent circuit.

Table 2 summarizes the results, where the results obtained from the conventional modelling routine are also listed for comparison. Both optimization routines were started from the same set of initial element values all of which were at least two times as great as the true values. Both optimizations were terminated by the Super-Compact optimizer with "gradient termination", which means the gradients of an error function with respect to element values are less than a predetermined value.

It is clearly seen that the new modelling method provides better accuracy than the conventional method. The modelling error was less than 3 % for all the element values except for R_d and L_d which are the least influential to the FET characteris-

Table 2: Accuracy comparison of the new routine with the conventional routine

Element	Conventional Routine		True Value	New Routine	
	Modeling Error(%)	Value obtained		Value obtained	Modeling Error(%)
g_m (mS)	-9.3	45.4	50.	49.5	-0.94
τ (ps)	-0.6	2.98	3.0	3.02	+0.5
C_{gs} (pF)	-7.8	.234	.25	.250	-0.1
C_{ds} (pF)	-4.5	.0766	.08	.078	-1.4
C_{dg} (pF)	+5.6	.0264	.025	.0250	0
R_g (ohms)	-37.1	2.517	4.0	4.03	+0.6
R_s (ohms)	-21.7	3.13	4.0	3.94	-1.4
R_d (ohms)	-1.8	2.95	3.0	2.49	-17.0
R_{ds} (ohms)	+12.4	281.	250.	251.	+0.5
L_g (pH)	+12.4	67.7	60.0	60.0	-0.1
L_d (pH)	+68.0	42.0	25.0	28.8	+15.0
L_s (pH)	+53.3	23.0	15.0	15.5	+3.0

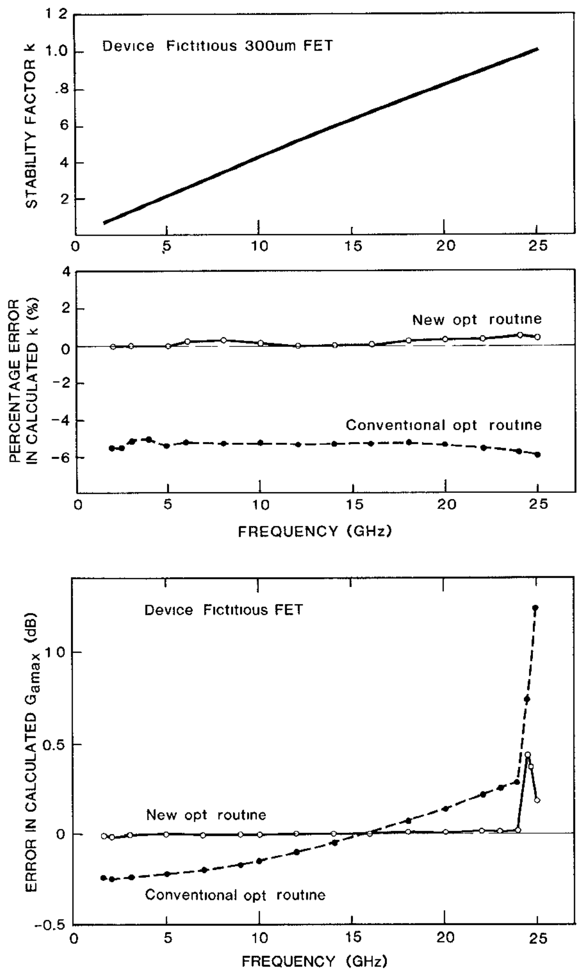

Fig. 2: Modelling error in (a) calculated stability factor, k, and in (b) calculated maximum available gain (when k > 1) or maximum stable gain. The modeled FET is the fictitious device shown in Table 2.

tics. Fig. 2 shows the modelling error in a stability factor, k, and in a maximum available gain, G_{Amax}. A maximum stable gain instead of G_{Amax} was calculated in the figures when $k < 1$. The new routine generated only less than 0.5% error in k and less than 0.02 dB error in G_{Amax}, as opposed to 5% and 0.3 dB errors by the conventional routine.

2) Application to Practical 0.5 um-gate MESFETs

The new modelling routine was applied to the modelling of three 0.5 um GaAs MMIC FETs fabricated in the Microwave Technology Division. They are two 10-finger 600 um FETs and one 14-finger 1200 um FET.

i) Measurement of S-parameters

The S-parameters of the first 600 um FET and the 1200 um FET were measured at frequencies from 1.5 GHz to 25 GHz using an on-wafer probing technique with a Cascade Model 42 probe station connected to an HP8510/8515 network analyzer system. A full 2-port error correction was performed at the probe tip terminals using Cascade-provided impedance standards.

The S-parameters of the other 600 um FET were measured on an HP8510 system using an in-house coplanar test fixture at frequencies from 1.0 GHz to 18 GHz. The device was mounted on a saphire-substrate chip carrier with coplanar transmission lines and was bonded using mesh wires. The carrier was then dropped into the fixture. A full 2-port error correction was performed at terminals inside the fixture using coplanar impedance standards.

ii) FET Modelling

Table 3 summarizes the results of modelling performed on the 1200 um FET. The modelling was accomplished with and without R_i optimized. In each case two completely different sets of initial element values were tested to examine the modelling consistency. The table also lists the results obtained by the conventional modelling routine for comparison. Superior consistency of the new routine is clearly seen in the table. The modelling without R_i optimized exhibits further improved consistency. This is because of substantial electrical equivalence of R_i to R_g, which tends to increase the degree of singularity of the associated normal equation if both are tried to optimize simultaneously. The superior consistency of the new routine is also demonstrated in Fig. 3, where G_{Amax} predicted from the model is compared with the one calculated directly from the measured S-parameters. Similar results were obtained for the Cascade-probed 600 um FET, which are summarized in Table 4 and Fig. 4. For both FETs the new modelling routine achieved better than 10 % consistency for almost all the element values.

Table 5 shows the results of modelling performed on the other 600 um FET measured in the coplanar test fixture at three different drain voltages.

Table 3: Comparison of modelling consistency performed on a 1200 um 0.5 um-gate GaAs FET

IV1 = Set 1 of initial values, IV2 = Set 2 of initial values

Element	Conventional Routine			New Routine					
				with R_i			without R_i		
	IV1	IV2	Inconsistency (%)	IV1	IV2	Inconsistency (%)	IV1	IV2	Inconsistency (%)
g_m (mS)	143.	139.	3.2	142.	139.	2.0	141.	140.	0.7
τ (ps)	2.77	2.84	2.5	2.55	2.66	4.2	2.61	2.64	1.0
C_{gs} (pF)	1.06	1.01	4.7	1.04	1.04	0	1.05	1.05	0
C_{ds} (pF)	.218	.212	2.8	.214	.206	3.8	.211	.207	1.6
C_{dg} (pF)	.0854	.0881	3.1	.0864	.0868	0.5	.0855	.0860	0.6
R_g (ohms)	.866	1.57	58.0	1.45	1.72	17.0	1.80	1.82	0.7
R_i (ohms)	.398	.024	117.	.486	.100	130.	----	----	----
(R_g + R_i)	(1.26)	(1.59)	23.0	(1.94)	(1.82)	6.1	----	----	----
R_s (ohms)	1.25	1.21	3.7	1.22	1.24	2.0	1.30	1.28	1.8
R_d (ohms)	1.06	.862	20.3	1.06	.548	63.0	.762	.55	32.0
R_{ds} (ohms)	64.5	65.4	1.4	66.8	67.7	1.3	66.7	67.2	0.6
L_g (pH)	6.98	13.0	60.5	9.91	9.85	0.6	10.1	9.85	2.7
L_d (pH)	7.89	0.62	170.8	13.7	16.2	16.9	14.7	15.8	6.9
L_s (pH)	7.18	8.59	17.9	7.87	8.50	7.7	8.29	8.43	1.7

As was expected, almost invariant values of L_g, L_s and L_d were observed while the values of g_m, C_{gs} and τ changed with drain voltage.

V. Conclusion

A new modelling algorithm has been developed which generates an accurate and consistent microwave FET model from measured S-parameters. The combinations of properly selected circuit elements with an appropriate S-parameter to be fitted greatly improves modelling accuracy. The separate use of the four S-parameters avoids ambiguity in estimating the variance of measurement errors in individual S-parameters.

An FET modelling performed on a fictitious measured data indicated better than 3% accuracy in determining the value of all the major elements in an FET equivalent circuit. The application of the new modelling routine to practical MMIC FETs demonstrated the modelling consistency of better than 10%.

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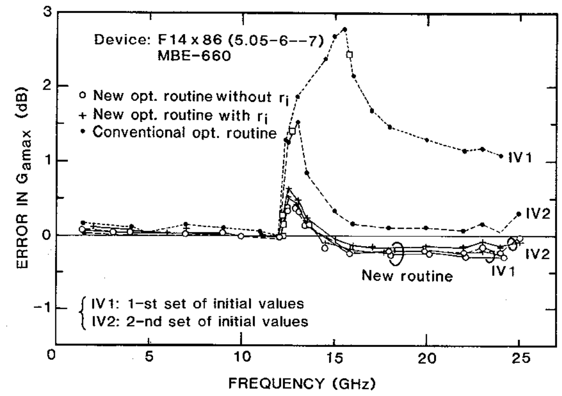

Fig. 3: Modelling errors in calculated maximum available gain (when k > 1) or maximum stable gain.
Device: 1200um MMIC 0.5um-gate GaAs FET

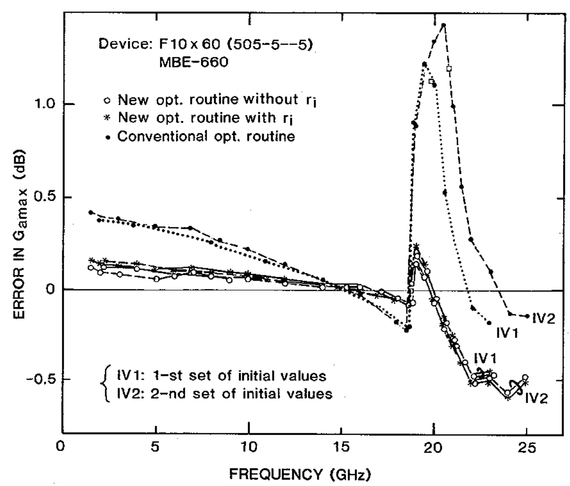

Fig. 4: Modelling errors in calculated maximum available gain (when k > 1) or maximum stable gain.
Device: 600um MMIC 0.5um-gate GaAs FET

Table 4: Comparison of modelling consistency performed on a 600 um 0.5 um-gate GaAs FET

IV1 = Set 1 of initial values, IV2 = Set 2 of initial values

Element	Conventional Routine			New Routine					
				with R_i			without R_i		
	IV1	IV2	Inconsistency (%)	IV1	IV2	Inconsistency (%)	IV1	IV2	Inconsistency (%)
g_m (mS)	100.	98.9	1.44	88.7	87.9	0.9	88.8	87.8	1.2
τ (ps)	1.90	1.67	12.7	1.87	1.84	1.6	1.88	1.88	0
C_{gs} (pF)	.631	.600	5.0	.554	.546	1.4	.557	.552	0.8
C_{ds} (pF)	.111	.118	6.0	.105	.105	0	.105	.104	0.8
C_{dg} (pF)	.0482	.0465	3.6	.0524	.0533	1.7	.0525	.0527	0.4
R_g (ohms)	2.35	1.78	27.4	2.30	2.04	12.2	2.48	2.48	0
R_i (ohms)	.263	.641	84.0	.268	.664	85.0	----	----	----
$(R_g + R_i)$	(2.61)	(2.43)	(7.4)	(2.57)	(2.70)	(4.9)	(2.48)	(2.48)	(0)
R_s (ohms)	3.00	3.06	2.0	2.40	2.27	5.7	2.44	2.40	1.4
R_d (ohms)	1.88	3.13	50.0	1.35	1.58	15.7	1.18	1.18	0
R_{ds} (ohms)	111.	105.	5.5	119.	121.	1.3	120.	121.	0.9
L_g (pH)	10.9	29.7	92.7	29.3	29.2	0.6	28.8	29.2	1.3
L_d (pH)	14.3	3.72	117.0	11.2	10.2	9.2	11.8	12.9	8.9
L_s (pH)	.81	1.06	27.0	6.86	6.97	1.6	7.28	7.61	4.4

Table 5
FET model of a 600 um FET biased at three different drain voltages (Device SR269-35A#7.04, V_G=-0.7V)

	Drain Voltage V_D		
	2V	4V	6V
g_m (mS)	65.9	62.7	58.9
τ (ps)	2.62	3.18	3.75
C_{gs} (pF)	.432	.466	.493
C_{ds} (pF)	.140	.142	.143
C_{dg} (pF)	.0785	.0719	.0695
R_g (ohms)	1.28	1.45	1.71
R_s (ohms)	1.56	1.83	2.02
R_d (ohms)	.463	.462	.260
R_{ds} (ohms)	86.1	115.	140.
L_g (pH)	171.	170.	169.
L_d (pH)	139.	135.	143.
L_s (pH)	19.9	19.8	19.7